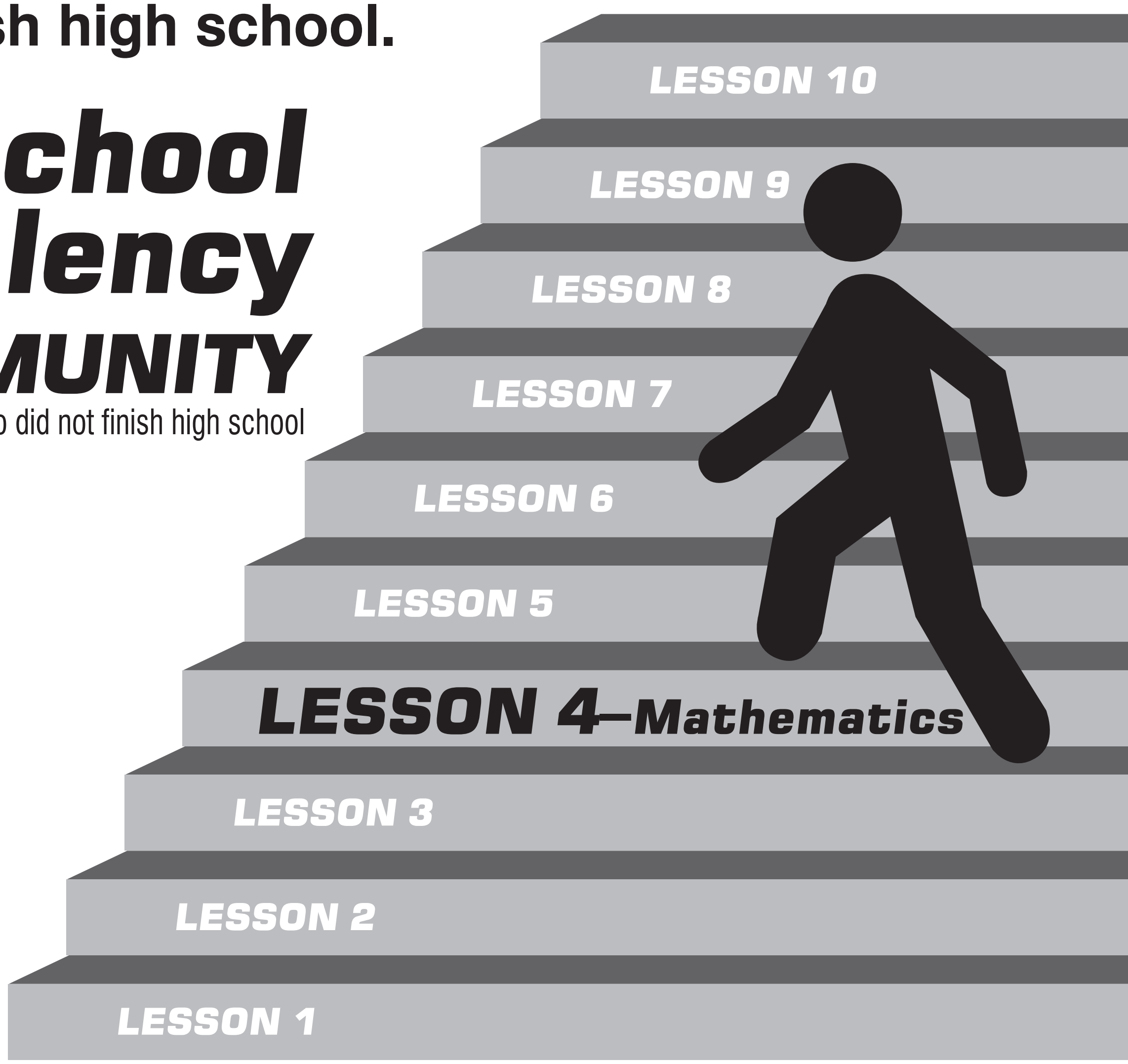
Steps to Success

There's never been a better time to finish high school.

High School Equivalency in the Community

a 'Mail-In' program for adults who did not finish high school



Fourth Step

FORGE AHEAD!

LESSON 4
Mathematical Reasoning

Vocabulary to Know

Equation–An equation is a math sentence with an equal sign. The left side of the equation has the same value as the right side of the equation. *Example:* $2+4 = 6$

Variable–A symbol for a number that you don't know yet. It is usually a letter like ***x*** or ***y***. *Example:* $x + 4 = 6$. In this equation ***x*** is the variable.

Linear Equation–An equation with just a variable like ***x*** rather than something more complicated like x^2 or x_{y2}

Linear equations are the simplest *equations* that you'll deal with. You've probably already solved linear equations; you just didn't know it. When you were learning addition, your teacher probably gave you worksheets to complete that had exercises like the following:

FILL IN THE BOX: ☐ + 3 = 5

Once you'd learned your addition facts well enough, you knew that you had to put a "2" in the box. Solving equations works in much the same way, but now you have to figure out what goes into the *x*, instead of what goes into the box. However, the equations can be much more complicated, so you will need a method.

In general, to solve an equation for a given variable, you need to "undo" whatever has been done to the variable. You do this in order to get the variable by itself; in technical terms, you are "isolating" the variable. This results in "(variable) equals (some number)", where (some number) is the answer they're looking for. *For instance:* **SOLVE: $x + 6 = -3$**

You want to get the *x* by itself; that is, you want to get "*x*" on one side of the "equals" sign, and some number on the other side. Since *x* needs to be on one side by itself, this means that plus 6 need to be removed. Because the 6 is *added* to the *x*, you need to *subtract* to get rid of it. That is, you will need to subtract a 6 from the *x* in order to "undo" having added a 6 to it.

This brings up the most important consideration with equations: No matter what kind of equation you're dealing with—linear or otherwise —whatever you do to the one side, you must do the exact same thing to the other side! Equations are like toddlers in this respect: You have to be totally, *totally* fair!

Whatever you do to an equation, do the SAME thing to BOTH sides of that equation!

Probably the best way to keep track of this subtraction of 6 from both sides is to format your work the way it is shown in the box.

$$\begin{array}{rcr} x + 6 & = & -3 \\ -6 & & -6 \\ \hline x & = & -9 \end{array}$$

What you see here is that 6 has been subtracted from both sides of the equation, then I have drawn an "equals" bar underneath both sides. 6 minus 6 is zero, and –3 –6 is –9 (remember the rules for adding and subtracting integers). The solution is the last line of my work: **$x = -9$**. Check your answer: $-9 + 6 = -3$

The same "undo" procedure works for subtraction: **Solve $x – 3 = -5$**

Since "*x*" needs to be by itself, you don't want the "-3" on the same side of the equation as the variable. The opposite of subtraction is addition, so you'll undo the -3 by adding 3 to both sides, and then adding down:

$$\begin{array}{rcr} x - 3 & = & -5 \\ +3 & & +3 \\ \hline x & = & -2 \end{array}$$

Then the solution is **$x = -2$**. Check your answer: $-2 – 3 = -5$

Now you try!

LESSON 4
Mathematical Reasoning

ASSIGNMENT 1

DIRECTIONS

Solve for the variable x. Show your work. Refer back to the examples provided.

1. $x - 4 = 5$ ______________________

2. $x - 12 = 24$ ______________________

3. $x + 1.7 = 3.3$ ______________________

4. $x + ¼ = ½$ ______________________

The "undo" of multiplication is division. If something is multiplied on the *x*, you undo it by dividing both sides (that is, dividing each term on both sides) of the equation by whatever is multiplied on the *x*: **SOLVE: 2*x* = 5**

Since the *x* is multiplied by 2, I need to divide both sides by 2:

$$2x = 5$$
$$\frac{2x}{2} = \frac{5}{2}$$
$$x = \frac{5}{2}$$

Then the solution is *x* = 5/2 or *x* = 2.5.

The "undo" of division is multiplication:
SOLVE: x/5 = -6

Since the *x* is divided by 5, you'll want to multiply both sides by 5:

$$\frac{x}{5} = -6$$
$$\frac{x}{5}\left(\frac{5}{1}\right) = -6(5)$$
$$x = -30$$

Then the solution is *x* = -30.

In the above solution, 5 is multiplied on the right-hand side of the equation, and by 5/1 on the left-hand side since 5 = 5/1. Why? It is often easier to keep track of what you're doing, when working with fractions if all the numbers involved are in fractional form.

There is one "special case" related to the "undoing multiplication" case above: When *x* is multiplied by a fraction, you "undo" this multiplication by dividing both sides of the equation by that fraction. To divide by a fraction, you flip-n-multiply. To isolate a variable that is multiplied by a fraction, just multiply both sides of the equation by the flip ("reciprocal") of that fraction. *For example:*

SOLVE: 3/5 x = 10

Since *x* is multiplied by 3/5, you'll want to multiply both sides by 5/3, to cancel off the fraction on the *x*. Many students find it helpful to also turn the 10 into a fraction, by putting it over 1.

$$\frac{5}{3}\left(\frac{3}{5}x\right) = \left(\frac{10}{1}\right)\frac{5}{3}$$
$$x = \frac{50}{3}$$

Then the solution is *x* = 50/3

Now you try!

LESSON 4
Mathematical Reasoning

DIRECTIONS

Solve for the variable. Use the previous examples. *Show your work on a separate sheet of paper*.

5. 4x = 5 ______________________________

6. 4x = 12 ______________________________

7. x/3 = 3______________________________

8. x ÷ 5 = 25 ______________________________

Most linear equations require more than one step for their solution. *For instance:*

SOLVE: 7x + 2 = -54

You need to undo the "times seven" and the "plus two". There is no rule about which "undo" should be done first. However, it is strongly suggested to do addition/subtraction before any multiplication/division.

$$
\begin{aligned}
7x + 2 &= -54 \\
-2 \quad & \quad -2 \\
\hline
\frac{7x}{7} &= \frac{-56}{7} \\
x &= -8
\end{aligned}
$$

Then the solution is x = -8

Check your answer: 7x + 2 = -54

7(-8) + 2 = -54
-56 + 2 = -54
-54 = -54

DIRECTIONS

Solve the following equations by finding the value of x. *Show your work on a separate sheet of paper and write your final answer in the correct squares below.* Look back at the examples to help you format your equations. Try to develop a way *you* solve equations. Use a consistent format.
Check your answers.

9. 3x = -12	**10.** x + 9 = -2	**11.** x + 8 = 3
12. -2 = x + 6	**13.** 6x = -24	**14.** x + 8 = 18
15. -9 + 6x = -81	**16.** 4x + 2 = 10	**17.** 5 + 5x = 45
18. -6x + 9 = 69	**19.** 9 + 3x = 27	**20.** 2x + 3 = -9

LESSON 4
Mathematical Reasoning

ASSIGNMENT 2

Equations can be written to represent word problems:

Maria bought 3 shirts. Each cost the same amount. If the total cost was $63, what was the cost of one shirt?

Use a variable to represent the unknown (the cost of one shirt). Let x = cost of one shirt. Write an equation (translate the word ***is*** to an equal sign). $3 \cdot x = 63$

DIRECTIONS

Write an equation for each sentence and solve. *Use a separate sheet of paper to show your work and answers*.

1. **A number less 4 is 12**
2. **The sum of a number and 4 is 12**
3. **A number divided by 4 is 12**
4. **Four times a number is 12**
5. **$5 less than the amount Steven has is $25**
6. **The hourly rate increased by $5 is $20.**
7. **Some money divided equally among 4 children is $52.**
8. **Ben pays $10 more than Clara for rent. Ben pays $320 for rent. What does Clara pay?**
9. **Kay bought 6 pens that were all the same price and a notepad for her desk that cost $2. She spent $11 in all. What is the cost of one pen?**
10. **A painter rented a wall paper steamer at 9 a.m. and returned it at 4 p.m. He paid a total of $28.84. What was the rental cost per hour?**

The Exponent

The ***exponent*** of a number says ***how many times*** to use the number in a multiplication.

In 8^2, the "2" says to use 8 twice in a multiplication, so $\mathbf{8^2 = 8 \times 8 = 64}$.

In words: 8^2 could be called "8 to the power 2"or "8 to the second power", or simply "8 squared."

Exponents are also called Powers or Indices.

Some more examples:

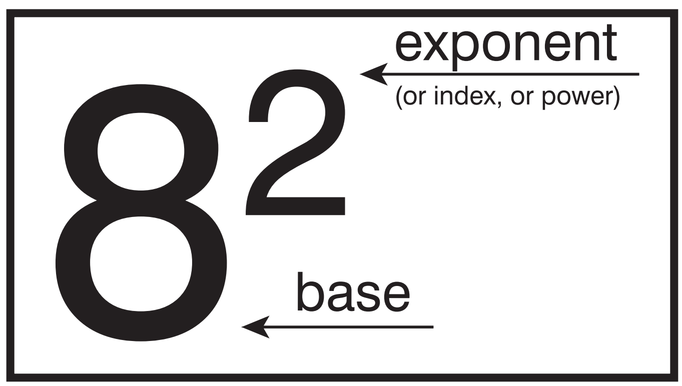
Example: $5^3 = 5 \times 5 \times 5 = 125$

In words: 5^3 could be called "5 to the third power," "5 to the power 3" or simply "5 cubed"

Example: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

In words: 2^4 could be called "2 to the fourth power," "2 to the power 4" or simply "2 to the 4th"

Exponents make it easier to write and use many multiplications.



LESSON 4
Mathematical Reasoning

ASSIGNMENT 3

DIRECTIONS
Solve for the value of the exponential notation.

1. $(-3)^3=$ ________________
2. $(-6)^2=$ ________________
3. $(-7)^2=$ ________________
4. $(10)^2=$ ________________
5. $(2)^3=$ ________________
6. $(9)^3=$ ________________
7. $(-4)^3=$ ________________
8. $(-12)^3=$ ________________
9. $(5)^3=$ ________________
10. $(12)^2=$ ________________

Graphs

In our daily life we come across numerical data in advertisements, newspapers and at other places. These data may relate to the cost of living, cricket average, profits of a company and so forth.

For example, if we look into a newspaper, we may find a weather report giving pertinent data about the maximum and minimum temperatures and rainfall of various important cities of a country.

Data recorded in experiments or surveys is displayed by a ***statistical graph***.

When you encounter a graph, do the following:

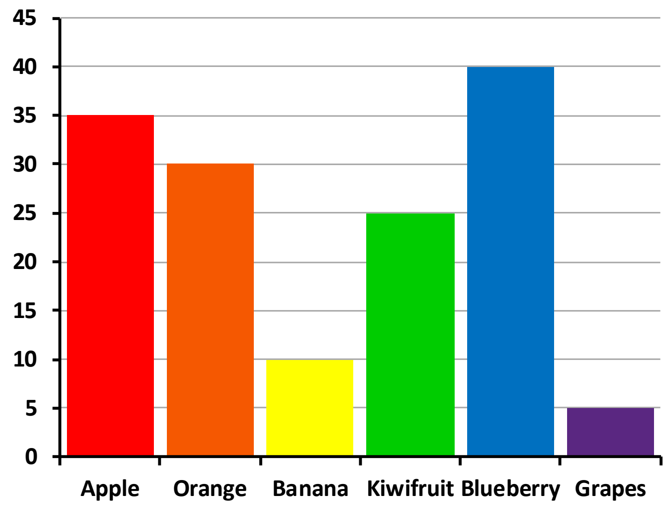
- Read the title of the graph. What information do you expect: What do you know about this subject already?
- Look for the measurements given along the bottom and side of the graph. Often, time moves from left (more distant past to right (more recent) on the bottom scale.
- Look for the source of the information.
- Finally, look at the graph itself and summarize what you see. In a circle graph, is there something that makes up a large wedge? In a line graph, is the trend upwards or downwards? In a bar graph, what is being compared or revealed? Are there changes or comparisons worth nothing?
- After you preview the graph, answer the questions about it.

Bar Graph–A bar graph is a pictorial representation of statistical data in which the independent variable attains only the discrete value. The dependent variable can either be discrete or continuous and is used to represent bars of different heights. It may be horizontal or vertical.

Bar graphs are probably the most popular and most widely used graph types.

The greater the length or height of a bar, greater will be their value.

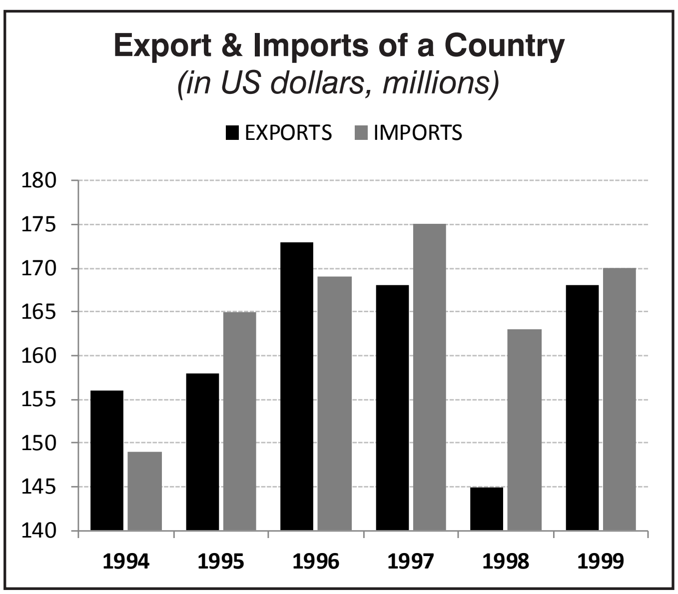
LESSON 4
Mathematical Reasoning



From the above graph, we see that Blueberry is most popular and Grapes are least popular.

ASSIGNMENT 4

DIRECTIONS
Answer the following questions according to the graph below.



1. **What is the name of the graph shown?** ____________________

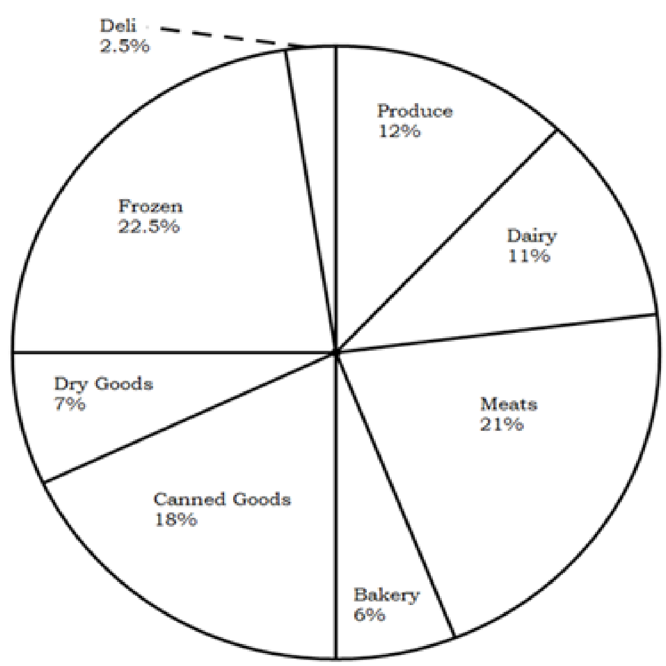
2. **Find the average (mean) *exports* for the given period.** ____________________

3. **What year shows the greatest amount of exports?** ____________________

4. **What year was the least amount of imports?** ____________________

Pie Chart–A pie chart is a special chart which uses "pie slices" to show relative sizes of data. It is a circular chart in which we divide the circle into sectors. They are used to compare different parts in the given data. The following pie chart shows: ***the breakdown of revenues for a particular grocery store over the first quarter of this year.***

Notice that the pie chart uses percentages. If you add all the slices together, you get 100%.



Total revenue, first quarter, $200,000

LESSON 4
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ASSIGNMENT 5

DIRECTIONS
Answer the following questions using the pie chart below.

Energy Use in the Average American Home

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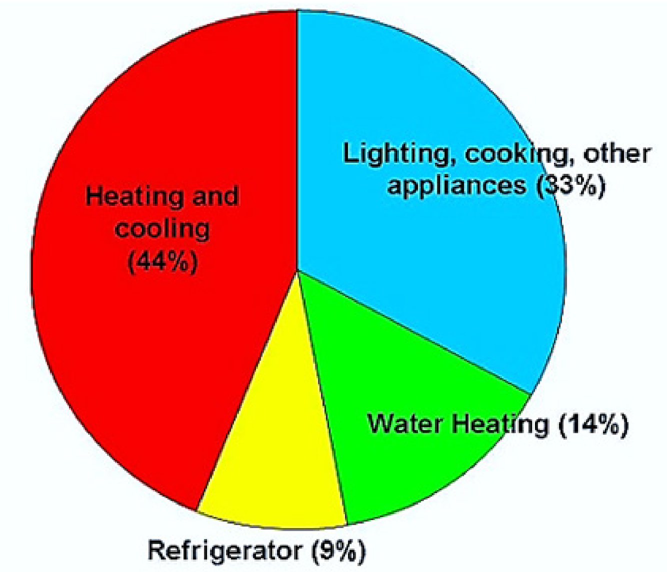


1. **The biggest energy consumers in a home are the heating and air conditioning systems, the water heater, and the refrigerator. According to the graph, these appliances account for what percentage of total energy usage?**
   A. 20%
   B. more than 20% and less than 60%
   C. more than 60% and less than 80%
   D. 80%
   E. more than 80%

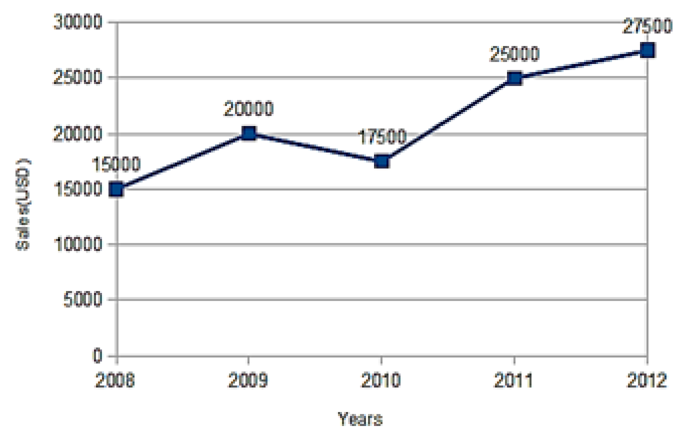
2. **If a family wants to reduce its energy bill(s), which system should they maintain well and use conservatively?**

   ____

The **Line Graph**, also known as a line chart, is one of the most common graphs. A line graph is a graph in which points belonging to an observation are plotted on an X-Y plane and every two consecutive points are joined together using a straight line.

Example of line graph:

Sales of a particular organization in different years



A line graph is often used to visualize rate of change of a quantity. It is more useful when the given data has peaks and valleys. Line graphs are very simple to draw and quite convenient to interpret.

The data used in a line graph is generally a sample data. A Line graph is drawn between two sets of variables one of which is represented on X axis and other on Y axis. One variable is independent variable and usually marked at X axis, while the other variable is dependent variable and usually marked at Y axis.

A line graph is preferred when rate of change of quantity is observed.

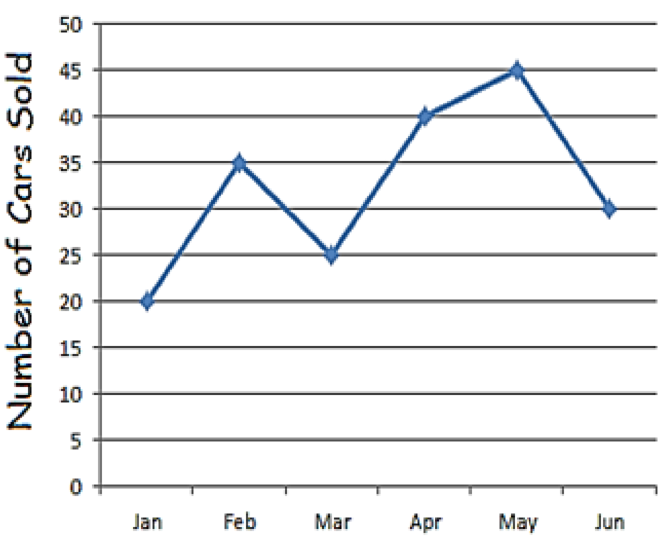
LESSON 4
Mathematical Reasoning

A few examples of the types of data used in line graph are as follows:

- Speed and time
- Sales and expenditure
- Year and production
- Days and number of an article sold
- Months and household expenditure
- States and their population
- City and temperature
- Height and weight
- Age and weight

ASSIGNMENT 6

DIRECTIONS
Use the line graph below to answer the following questions.

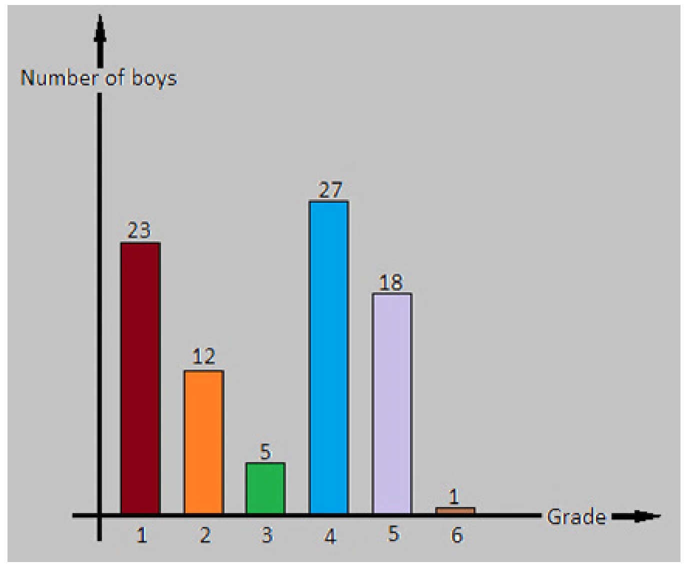


1. **Which month was the highest number of cars sold? ____________________**
2. **Which month was the lowest number of cars sold? ____________________**
3. **What is the range of cars sold from January to June? ____________________**
4. **How many months had less than 30 sells?**

____

ASSIGNMENT 7

DIRECTIONS
Use the graphs below to answer the questions.



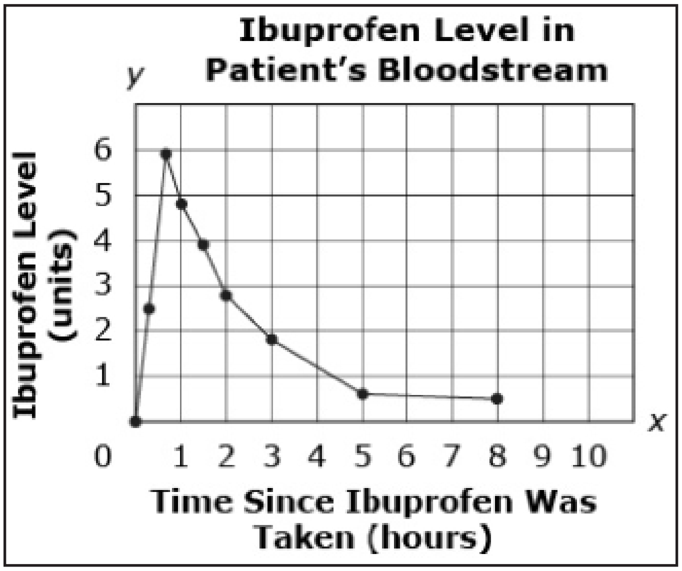
1. **Calculate the *mean, median, mode* and *range* of the number of boys in the given grades. Match the measures to their respective numerical values.**

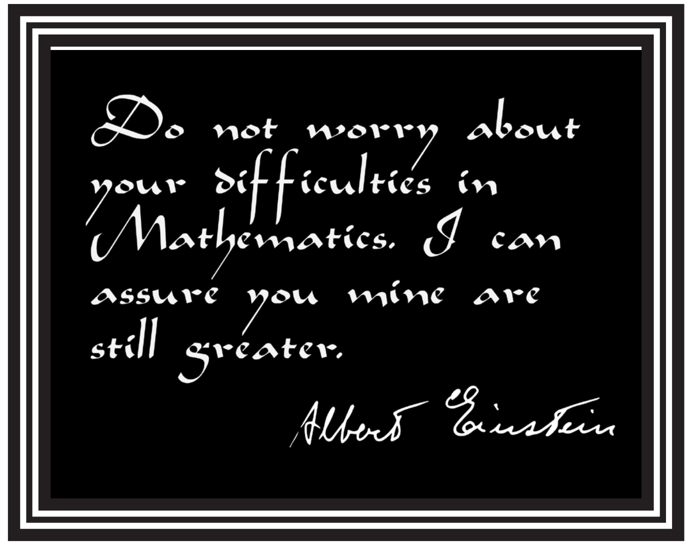
Mean ____________
Median ____________
Mode ____________
Range ____________

A. 14.3
B. 15
C. 26
D. none

LESSON 4
Mathematical Reasoning

The graph a shows the level of ibuprofen, y units, in a patient's bloodstream, x hours, after the ibuprofen was taken.





2. Fill in the blanks. Select your answers from the table. You have selections for blank (a) and blank (b).

According to the graph the level of ibuprofen in the patient's bloodstream increased from (a)______ hour to (b)______ hours.

Choices for (a)	Choices for (b)
0	2/3
2/3	2½
2½	5
5	6
8	8

References

www.ple.plato.com
www.purplemath.com
www.mathcaptain.com